# Design of Surface-Mounted Permanent Magnet Vernier Machines Considering Harmonic Characteristics of Winding MMF

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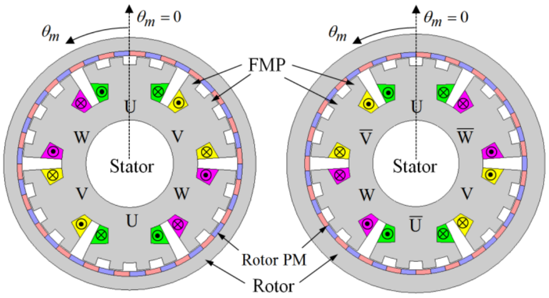
**This paper presents a design method of surface-mounted permanent magnet vernier (SPMV) machines considering the harmonic characteristics of the winding magneto-motive force (MMF). Based on the magnetic gearing effects, the armature magnetic field in the SPMV machine is modulated by the flux modulation poles (FMP) which are formed in the stator. Particularly, the winding MMF harmonics can affect working harmonic of the air-gap flux density distribution, which generates output torque on the rotor, according to the harmonic characteristics of the air-gap permeance. Thus, in this paper, the FMP shapes to improve the output torque are designed by using the analytical equations for the winding MMF, permeance and flux density in the air-gap. In the SPMV machines with same numbers of the slot and FMPs, but the different winding configurations, the FMP shapes to maximize the working harmonic are different according to harmonic characteristics of the winding MMF. Conseguently, thier effects on the working harmonic and output torque are also differnet. In addition, it can be found that the effects of the design for the FMP shape on the torque increase ratio are determined according to the winding configurations in the SPMV machine.**

***Index Terms*— Armature, electromagnetic devices, magnetic flux density, permanenet magnet motors.**

## I. INTRODUCTION

IN low-speed direct drive applications, surface-mounted permanent magnet vernier (SPMV) machines are considered as the good choice due to their well known advantages such as the high torque density and high efficiency [1]. In the SPMV machine, the magneto-motive force (MMF) of the armature windings is modulated by the flux modulation poles (FMPs) which are formed on the stator. And then, the modulated armature magnetic field generates torque by interacting with the magnetic field of the rotor PMs. It indicates that the torque capacity of the SPMV machine depends on the working harmonic of the modulated armature field that corresponds to the magnetic poles of the rotor PMs. Unlike the regular PM machine, the winding MMF harmonics of the SPMV machine can generate the working harmonic and output torque by adjusting the harmonic characteristics of the air-gap permeance [2]. Thus, to improve the working harmonic, the FMP shapes should be designed by considering the harmonic characteristics of the winding MMF and air-gap permeance. This paper presents the design method for the FMP shapes in the SPMV machines according to the winding configurations. The analytical expressions for the winding MMF, permeance and flux density in the air-gap are derived by the Fourier analysis method [3]. And then, each FMP shape for the SPMV machines with two different winding configurations is designed by using the analytical equations. When the number of the slots is 6, the two armature winding configurations produce the magnetic field of 2 and 4 poles, respectively. In both machines, the working harmonic is increased by designing the shapes of the FMPs based on the analytical equations. In addition, by using the finite element analysis (FEA) method, it is verified that the increase ratio of the working harmonic has a same trend to the torque increase ratio. The analysis results show that the torque increase ratio is higher in the SPMV machine with the armature magnetic field of 4 poles.



(a) (b)

Fig. 1. Structures of the SPMV machines with the three-phase concentrated windings (a) Machine I (b) Machine II.

## II. SPMV MACHINES

### A. Winding Configurations

Fig. 1 shows the structures of the SPMV machines with the three-phase concentrated windings. In common, the stators of the SPMV machines have 6 slots and each tooth is split into four FMPs. However, the armature windings in Machine I and II produce the magnetic field of 4 and 2 poles, respectively. Based on the following rule for the normal operation of the SPMV machines, the rotor pole pairs, $p_r$ for Machine I and II are set by the 22 and 23, respectively [1].

$$p_r = N_{FMP} - p_s \tag{1}$$

where, $p_s$ and $N_{FMP}$ are the number of the stator pole pairs and FMPs, respectively. In both machines, the basic specifications such as the amounts of the PMs, the number of turns and input current are same. Moreover, the dimensions except for the thickness of the rotor yoke are applied identically. Owing to the magnetic saturation, the thickness of the rotor yoke in Machine II is increased twice compared with that of Machine I.

### B. FMP Design and Output torque

Fig. 2 shows the winding MMF distributions and harmonic components for the Machine I and II. Using the Fourier analysis method, the winding MMF distributions can be expressed as

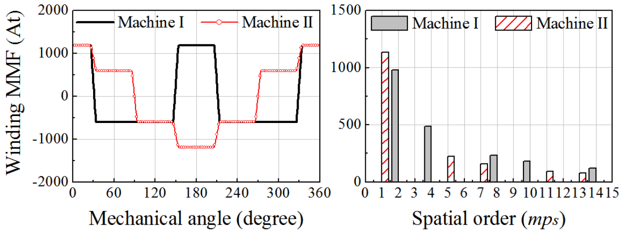


Fig. 2. Winding MMF distributions and the harmonic components.

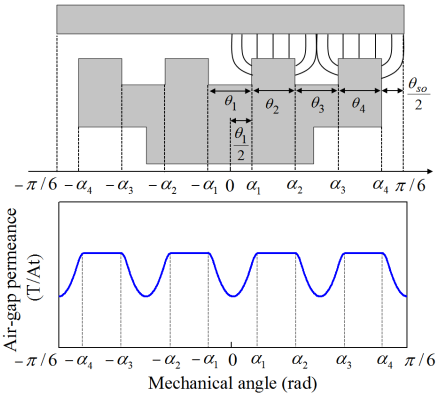


Fig. 3. Design variables and the air-gap permeance distribution.

$$F(\theta_m,t)=\sum_{m=1,4,7,\cdots}^{\infty}F_{mp_s}\cos(mp_s\theta_m-\omega_e t) \tag{2}$$
$$+\sum_{m=2,5,8,\cdots}^{\infty}F_{mp_s}\cos(mp_s\theta_m+\omega_e t)$$

where $\theta_m$ is the mechanical angular position and $F_{mps}$ is the Fourier coefficient for the $(mp_s)^{th}$ spatial harmonic of the winding MMF. Fig. 3 shows the design variables for the FMP shapes formed on the tooth. In addition, the air-gap permeance distribution is given as shown in Fig. 3 and is expressed as

$$P(\theta_m)=P_0+\sum_{j=1,2,3,\cdots}^{\infty}P_j\cos(j\theta_m) \tag{3}$$

where $P_0$ is the DC offset value, and $P_j$ is the Fourier coefficient for the $j^{th}$ spatial harmonic of the air-gap permeance function. In addition, the $P_0$ and $P_j$ are given as the functions of the design variables for the FMP shapes.

By multiplying (2) and (3), the air-gap flux density distribution by the armature windings in the air-gap can be obtained. For the Machine I and II, the working harmonic components are arranged as followings [2].

- For Machine I,

$$B^{22nd}(\theta_m,t)=B_{22}\cos(22\cdot\theta_m+\omega_e t) \tag{4}$$

$$\begin{aligned}B_{22}=\big[&F_{22}P_0+0.5\times(F_2P_{24}+F_4P_{18}+F_8P_{30}+F_{10}P_{12}\\&+F_{14}P_{36}+F_{16}P_6+F_{20}P_{42}+\cdots)\big]\end{aligned} \tag{5}$$

- For Machine II,

$$B^{23th}(\theta_m,t)=B_{23}\cos(23\cdot\theta_m+\omega_e t) \tag{6}$$

$$\begin{aligned}B_{23}=\big[&F_{23}P_0+0.5\times(F_1P_{24}+F_5P_{18}+F_7P_{30}+F_{11}P_{12}\\&+F_{13}P_{36}+F_{17}P_6+F_{19}P_{42}+\cdots)\big]\end{aligned} \tag{7}$$

It indicates that all the winding MMF harmonics can affect the working harmonic according to harmonic characteristics of the air-gap permeance function. Since $F_{mps}$ and $P_j$ are expressed as the functions of the design variables for the FMP shape, the dimensions for the design variables to maximize the working harmonic in the Machine I and II can be calculated by (5) and (7), resepctively. The design results for the Machine I and II are listed in Table I and Table II, respectively. By applying the improved FMP shapes, the working harmonics in the Machine I and II are increased by 26% and 11%, respectively. In both machines, the increase ratio of the average torque has a same trand to the increased ratio of the working harmonic. The different effects of the designed FMP shape according to the winding configurations can be explained by analyzing the harmonic characteristics of the air-gap permeance as shown in Fig. 4. In Machine I, to improve the output torque by using the winding MMF harmonics, the $12^{th}$ and $18^{th}$ spatial harmonics of the air-gap permeance are increased. However, in the Machine II, the winding MMF distribution is close to a sinusoidal wave, and thus the $24^{th}$ harmonic of the air-gap permeance, which improve the working harmonic by using the fundamental wave of the winding MMF, is mainly increased. The analysis results shows that the torque density of the SPMV machine can be more improved when employing the winding configuration in Machine I. In addition, the proposed design method in this paper is useful to develop the winding configurations that are to increase the torque density of the SPMV machine.

TABLE I
DESIGN RESULTS FOR THE FMP SHAPE FOR MACHINE I

| FMP shape | Design variables [degree] | | | | | $\lvert B_{22}\rvert$ [T] | | Torque [Nm] |
|---|---|---|---|---|---|---|---|---|
| | $\theta_1$ | $\theta_2$ | $\theta_3$ | $\theta_4$ | $\theta_{so}$ | *Analytical* | *FEA* | |
| Basic | 7.5 | 7.5 | 7.5 | 7.5 | 7.5 | 0.034 | 0.033 | 47.4 |
| Improved | 9.2 | 6.3 | 12.1 | 4.4 | 5.5 | 0.043 | 0.042 | 59.5 |

TABLE II
DESIGN RESULTS FOR THE FMP SHAPE FOR MACHINE II

| FMP shape | Design variables [degree] | | | | | $\lvert B_{23}\rvert$ [T] | | Torque [Nm] |
|---|---|---|---|---|---|---|---|---|
| | $\theta_1$ | $\theta_2$ | $\theta_3$ | $\theta_4$ | $\theta_{so}$ | *Analytical* | *FEA* | |
| Basic | 7.5 | 7.5 | 7.5 | 7.5 | 7.5 | 0.034 | 0.033 | 46.9 |
| Improved | 11 | 4.5 | 11.4 | 3.6 | 10 | 0.037 | 0.036 | 52.1 |

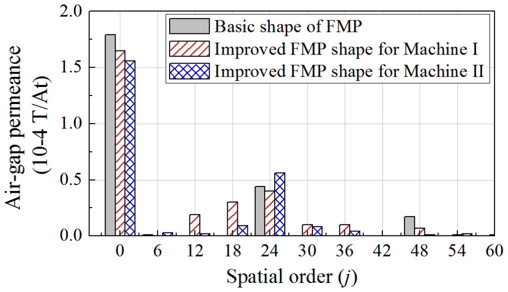


Fig. 4. Harmonic components of the air-gap permeance.

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